

# Robust Navigation System for a Land Vehicle

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**Abstract.** This paper presents the integration of Inertia Navigation System and Global Position System (GPS) using Unscented Kalman Filter (UKF). The nonlinear system model is used because linearized system models introduce errors in high dynamic environments. The navigation performance and robustness of the proposed algorithm are also compared with that of the extended Kalman filter (EKF). To enhance the navigation performance, the non-holonomic constraint is applied to the UKF and it is found that the robustness of system is better than before when the GPS signal outages.

**Keywords:** Integrated navigation, Non-holonomic constraint, robust navigation, UKF

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## INTRODUCTION

It is difficult to develop an accurate vehicle navigation using GPS as satellite signals cannot be guaranteed at all times. So, augmentation of GPS with INS requires to improve navigation accuracies. The main drawback of an INS is the degradation of its performance with time. In order to limit the errors to an acceptable level, regular updates are necessary and GPS measurements can be used to this purpose. The most commonly used integration schemes in literatures are loosely and tightly coupled integration strategy. The tightly coupling has better performance in urban or natural canyons because it can provide an integrated navigation solution also with less than four satellites [1],[2].

In a tightly coupled GPS/INS system, the system dynamic models as well as the measurement models are nonlinear. Therefore, the nonlinear system and the measurement models are simply linearized around the current state estimate to apply the usual EKF. EKF techniques suffer from divergence during GPS outages when using low-cost IMUs due to approximations during linearization process and suboptimal modeling. The main reason is that the low-cost sensors have complex error characteristics which are stochastic in nature and difficult to model. There are two main types of model for INS: error state space model and total state space model [1]. The error state space model is special cases of total state space model and can be obtained using linearization techniques. Although these error models save computing time, the performance and robustness decreases because of error introduced by linearization. To overcome these drawbacks and to enhance the performance and

robustness of the tightly coupled integration, instead of an EKF, the UKF is applied on nonlinear total state model in this work.

While the EKF approximates the propagation of mean and covariance of stochastic variables through nonlinear system dynamic and measurement models by linear transformations, which is accurate to first order only, in UKF a set of specifically selected sigma-points is propagated through the nonlinear models. Mean and covariance are then calculated from the set of transformed sigma-points, which is accurate to second order [3]. The navigation accuracy of INS is also improved by applying the non-holonomic constraint (NHC) to the GPS/INS integration and it provides the continuous navigation solution even during GPS signal outages [4].

The total state non-linear model and error state perturbation model of INS, and measurement models for pseudorange and pseudorange rate observations are described in section 2, and the computing steps of UKF are briefly explained in section 3. In section 4, the NHC and how to use it in UKF are discussed. Simulation results are presented in section 5 and conclusions are reported in the last section.

## TIGHT INTEGRATION OF GPS/INS

In the tightly coupled integration scheme, INS data and GPS raw measurements (pseudorange and pseudorange rate) are processed in the data fusion algorithm, and the estimated errors are feedback to the INS to prevent the growth of navigation errors with time exhibited by an unaided INS [5]. The system dynamic models and the pseudorange and pseudorange

rate measurement models are the key to the development of GPS/INS data fusion algorithms.

### System Models

The total state INS mechanization model is given by the following differential equations [1],[2]. In this work, navigation frame mechanization was chosen.

$$\dot{\mathbf{L}} = \frac{v_N}{R_m+h}, \quad \dot{l} = \frac{v_E}{(R_t+h)\cos L}, \quad \dot{h} = -v_D \quad (1)$$

$$\dot{\mathbf{v}}^n = \mathbf{C}_b^n \mathbf{f}^b - (2\boldsymbol{\omega}_{ie}^n + \boldsymbol{\omega}_{en}^n) \times \mathbf{v}^n + \mathbf{g}^n \quad (2)$$

The attitude quaternion is propagated by

$$\dot{\mathbf{q}} = \frac{1}{2} \boldsymbol{\Omega}(\boldsymbol{\omega}_{nb}^b) \mathbf{q} \quad (3)$$

$$\text{where } \boldsymbol{\Omega}(\boldsymbol{\omega}_{nb}^b) = \begin{bmatrix} -[\boldsymbol{\omega}_{nb}^b \times] & \boldsymbol{\omega}_{nb}^b \\ -(\boldsymbol{\omega}_{nb}^b)^T & 0 \end{bmatrix}$$

and  $\boldsymbol{\omega}_{nb}^b$  is angular rate of a body frame relative to navigation frame.

Since the above nonlinear equations cannot be applied to EKF, they are linearized by perturbation and the linearized error state model is given by

$$\delta \dot{\mathbf{r}} = \delta \mathbf{v} \quad (4)$$

$$\delta \dot{\mathbf{v}}^n = [\mathbf{C}_b^n \mathbf{f}^b] \times \boldsymbol{\phi} + \mathbf{C}_b^n \delta \mathbf{f}^b - (2\boldsymbol{\omega}_{ie}^n + \boldsymbol{\omega}_{en}^n) \delta \mathbf{v}^n - (2\delta \boldsymbol{\omega}_{ie}^n + \delta \boldsymbol{\omega}_{en}^n) \times \mathbf{v}^n \quad (5)$$

$$\dot{\boldsymbol{\phi}} = -\boldsymbol{\omega}_{in}^n \times \boldsymbol{\phi} - \mathbf{C}_b^n \delta \boldsymbol{\omega}_{ib}^b + \delta \boldsymbol{\omega}_{in}^n \quad (6)$$

$\delta \mathbf{f}^b$  and  $\delta \boldsymbol{\omega}_{ib}^b$  can be defined as  $\delta \mathbf{f}^b = \nabla + \mathbf{v}_a$  and  $\delta \boldsymbol{\omega}_{ib}^b = \boldsymbol{\varepsilon} + \mathbf{v}_g$  where  $\nabla = [\nabla_x \nabla_y \nabla_z]^T$  and  $\boldsymbol{\varepsilon} = [\varepsilon_x \varepsilon_y \varepsilon_z]^T$  are biases in accelerometer and gyro outputs, and  $\mathbf{v}_a$  and  $\mathbf{v}_g$  are white Gaussian noise corrupting the measurements[6]. For UKF implementation, the total state model (equation (1), (2) and (3)) is used and the state vector is defined as

$$\mathbf{x} = [L \ l \ h : v_N \ v_E \ v_D : q_0 \ q_1 \ q_2 \ q_3 : \nabla_x \ \nabla_y \ \nabla_z : \varepsilon_x \ \varepsilon_y \ \varepsilon_z : cb \ d]^T$$

where  $[q_0 \ q_1 \ q_2 \ q_3]$  is the attitude quaternion vector. Equation (4), (5) and (6) are only used in EKF [7] and its state vectors is

$$\mathbf{x} = [\delta L \ \delta l \ \delta h : \delta v_N \ \delta v_E \ \delta v_D : \phi_N \ \phi_E \ \phi_D : \nabla_x \ \nabla_y \ \nabla_z : \varepsilon_x \ \varepsilon_y \ \varepsilon_z : cb \ d]^T$$

Latitude, longitude and height above the ellipsoid are denoted as  $L$ ,  $l$  and  $h$  and  $v_N$ ,  $v_E$  and  $v_D$  are velocities components in navigation frame respectively. The accelerometer bias and gyro bias are modeled as random constants and the clock bias  $cb$  in meter and clock drift  $d$  in meter per seconds of GPS receiver are calculated using random walk model [4].

### The Observation Models

The observables in our integration are pseudorange and pseudo-range rate of a GPS receiver. The

pseudorange measurement  $\rho_i$  to the  $i^{\text{th}}$  satellite can be modeled as follows:

$$\rho_i = \sqrt{(\mathbf{r}_i^n - \mathbf{C}_b^n \mathbf{l}_b)^T + (\mathbf{r}_i^n - \mathbf{C}_b^n \mathbf{l}_b)} + cb + v_{\rho,i} \quad (7)$$

The satellite position in n-frame coordinates is denoted with  $\mathbf{r}_i^n$  while the position of the GPS antenna is  $\mathbf{C}_b^n \mathbf{l}_b$ .  $\mathbf{l}_b$  is the lever arm vector pointing from the origin of the body frame defined by the IMU to the GPS antenna. Additionally, the receiver clock bias  $cb$  and the measurement noise  $v_{\rho,i}$  are included. The pseudorange rate can be described by

$$\dot{\rho}_i = \mathbf{e}_i^{nT} (\mathbf{v}_i^n - \mathbf{v}_{ins}^n - \mathbf{C}_b^n \boldsymbol{\omega}_{eb}^b \times \mathbf{l}_b) + d + v_{r,i} \quad (8)$$

$\mathbf{e}_i^n$  is the unit vector pointing from the GPS antenna to the  $i^{\text{th}}$  satellite and  $\boldsymbol{\omega}_{eb}^b$  the rate of body frame relative to the earth frame. Additionally, measurement noise  $v_{r,i}$  and the clock error drift  $d$  enter this observation model. Although the equation (7) and (8) can be used directly for UKF, the linearized model given in [2] must be used for EKF implementation.

### UNSCENTED KALMAN FILTER

UKF does not require to approximate nonlinear system dynamic and measurement models using the Jacobian in order to calculate the covariance of a random vector propagated through the nonlinear models. But a set of deterministically selected sigma-points which have the same mean and covariance as the original random vector is chosen. Then, these sigma-points are propagated through the nonlinear models, and the mean and the covariance of the transformed random vector is calculated from the propagated sigma-points [3]. Then, given an covariance matrix  $\mathbf{P}_k$ , the set of  $(2n+1)$  sigma points  $\boldsymbol{\chi}_k \in \mathfrak{R}^{2n \times 1}$  is computed as follows:

$$\begin{aligned} \boldsymbol{\chi}_k^0 &= \hat{\mathbf{x}}_k \\ \boldsymbol{\chi}_k^i &= \hat{\mathbf{x}}_k + \gamma \sqrt{\mathbf{P}_k} \quad \text{for } i=1 \dots n \\ \boldsymbol{\chi}_k^i &= \hat{\mathbf{x}}_k - \gamma \sqrt{\mathbf{P}_k} \quad \text{for } i=n+1 \dots 2n \end{aligned}$$

where  $\gamma = \sqrt{n + \lambda}$ , and  $\lambda = \alpha^2(n + \kappa) - n$  and it is a composite scaling parameter. The constant  $\alpha$  determines the spread of the sigma points around  $\hat{\mathbf{x}}_k$  and is usually set to a small positive value ( $0 \leq \alpha \leq 1$ ). The constant  $\kappa$  is a secondary scaling parameter which is usually set to 0 to 3 - n and provides an extra degree of freedom for fine tuning of the higher order moments [3]. The calculation steps for UKF are as follows:

1. Assign the sigma points

$$\boldsymbol{\chi}_k = [\hat{\mathbf{x}}_k \quad \hat{\mathbf{x}}_k + \gamma \sqrt{\mathbf{P}_k} \quad \hat{\mathbf{x}}_k - \gamma \sqrt{\mathbf{P}_k}]$$

where  $\sqrt{\mathbf{P}_k}$  is the Cholesky factorization of covariance matrix.

2. Propagate the sigma point using non-linear state equation.

$$\chi_{k+1}^i = f(\hat{\chi}_k, k)$$

3. Calculate the propagated mean and covariance of the state vector.

$$\bar{\mathbf{x}}_{k+1} = \sum_{i=0}^{2n} \mathbf{W}_m^i \chi_{k+1}^i$$

$$\mathbf{P}_{k+1}^{xx} = \sum_{i=0}^{2n} \mathbf{W}_c^i (\chi_{k+1}^i - \bar{\mathbf{x}}_{k+1}) (\chi_{k+1}^i - \bar{\mathbf{x}}_{k+1})^T$$

where

$$\mathbf{W}_m^0 = \frac{\lambda}{n+\lambda}$$

$$\mathbf{W}_c^0 = \frac{\lambda}{n+\lambda} + (1 - \alpha^2 + \beta)$$

$$\mathbf{W}_m^i = \mathbf{W}_c^i = \frac{\lambda}{2(n+\lambda)} \quad i = 1 \dots 2n$$

(In this work,  $\alpha = 0.99$  and  $\beta = 2$ .)

4. Calculate a set of predicted measurements by propagating sigma points through the nonlinear measurement model.

$$\mathbf{y}_{k+1}^i = h(\chi_{k+1}^i, k+1)$$

The mean and covariance are

$$\bar{\mathbf{y}}_{k+1} = \sum_{i=0}^{2n} \mathbf{W}_m^i \mathbf{y}_{k+1}^i \quad (9)$$

$$\mathbf{P}_{k+1}^{yy} = \sum_{i=0}^{2n} \mathbf{W}_c^i (\mathbf{y}_{k+1}^i - \bar{\mathbf{y}}_{k+1}) (\mathbf{y}_{k+1}^i - \bar{\mathbf{y}}_{k+1})^T + \mathbf{R}_{k+1}$$

$$\mathbf{P}_{k+1}^{xy} = \sum_{i=0}^{2n} \mathbf{W}_c^i (\chi_{k+1}^i - \bar{\mathbf{x}}_{k+1}) (\mathbf{y}_{k+1}^i - \bar{\mathbf{y}}_{k+1})^T + \mathbf{R}_{k+1}$$

$$\mathbf{K}_{k+1} = \mathbf{P}_{k+1}^{xy} (\mathbf{P}_{k+1}^{yy})^{-1}$$

5. Finally the estimated state is obtained by

$$\hat{\mathbf{x}}_{k+1}^+ = \bar{\mathbf{x}}_{k+1} + \mathbf{K}_{k+1} (\mathbf{y}_{k+1} - \bar{\mathbf{y}}_{k+1})$$

The covariance is updated as

$$\mathbf{P}_{k+1} = \mathbf{P}_{k+1}^{xx} - \mathbf{K}_{k+1} \mathbf{P}_{k+1}^{xy} \mathbf{K}_{k+1}^T$$

The UKF is an efficient derivative free filtering algorithm for computing approximate solutions to discrete-time non-linear optimal filtering problems. Since, in its original form, the UKF is a discrete-time

algorithm and it cannot be directly applied to continuous-time problems, where the state dynamics and measurement processes are modeled as continuous-time stochastic processes. In this work, although the system model is non-linear and continuous-time process, we consider the system as a discrete-time system in order to reduce the amount of computation because there is no significant difference between the continuous-time UKF and discrete-time UKF for the short time step [8]. In our calculation, each of the sigma point are integrated through the noise free dynamic model using 10 steps of the 4th ordered Runge- Kutta integration and then the mean and covariance are calculated. After that the measurement update is performed.

## NON-HOLONOMIC CONSTRAINT

NHC refers to the fact that unless the vehicle jumps off the ground (along z-axis) or slides on the ground (along y-axis), the velocity of the vehicle in the plane perpendicular to the forward direction (along x-axis) is almost zero [4]. Therefore, two NHCs can be considered as additional measurement updates in addition to the GPS pseudorange and pseudorange rate measurements to the UKF. According to this assumption

$$v_y^b \approx 0 + \varepsilon_{vy} \quad \text{and} \quad v_z^b \approx 0 + \varepsilon_{vz},$$

where  $\varepsilon_{vy}$  and  $\varepsilon_{vz}$  are the measurement noise value denoting any possible discrepancies in the above stated assumptions for a particular direction (x or z). The magnitude of the noise is chosen to reflect the extent of the expected constraint violations.

The NHC in body frame ( $v_y^b$  and  $v_z^b$ ) can be converted to navigation frame as

$$\hat{\mathbf{v}}^b = \hat{\mathbf{C}}_n^b \hat{\mathbf{v}}^n$$

For the UKF, the sigma points of velocity in navigation frame obtained from time update is pre-multiplied by  $\hat{\mathbf{C}}_n^b$  computed from quaternion sigma points to yield the body frame velocity.

$$\hat{\mathbf{v}}_{k+1}^{b,i} = \hat{\mathbf{C}}_n^b(\chi_{k+1}^i) \cdot \chi_{k+1}^{i,v^n}$$

where  $\chi_{k+1}^{i,v^n}$  is the sigma points represented by velocity in n-frame. Then the measurement update can be obtained by

$$\bar{\mathbf{v}}_{k+1}^b = \sum_{i=0}^{2n} \mathbf{W}_m^i \mathbf{v}_{k+1}^{yz,i}$$

where  $\mathbf{v}_{k+1}^{yz,i} = [\hat{\mathbf{v}}_{k+1}^{y,i} \hat{\mathbf{v}}_{k+1}^{z,i}]^T$  is  $x$  and  $y$  components of  $\hat{\mathbf{v}}_{k+1}^b$ .

$$\mathbf{P}_{k+1}^{yy} = \sum_{i=0}^{2n} \mathbf{W}_c^i (\mathbf{v}_{k+1}^{yz} - \bar{\mathbf{v}}_{k+1}^b) (\mathbf{v}_{k+1}^{yz} - \bar{\mathbf{v}}_{k+1}^b)^T + \mathbf{R}_{k+1}^v$$

$\mathbf{R}^v$  is measurement noise covariance matrix and it depends on the noise of forward velocity  $\hat{v}_{k+1}^x$  and misalignment angle.

$$\mathbf{P}_{k+1}^{xy} = \sum_{i=0}^{2n} \mathbf{W}_c^i (\mathbf{x}_{k+1}^i - \bar{\mathbf{x}}_{k+1}) (\mathbf{v}_{k+1}^{yz} - \bar{\mathbf{v}}_{k+1}^b)^T + \mathbf{R}_{k+1}$$

The estimated output is

$$\hat{\mathbf{x}}_{k+1}^+ = \bar{\mathbf{x}}_{k+1} + \mathbf{K}_{k+1} (-\bar{\mathbf{v}}_{k+1}^b)$$

If both NHC and GPS observables are used at the same time, the equation (9) becomes

$$\bar{\mathbf{y}}_{k+1} = \sum_{i=0}^{2n} \mathbf{W}_m^i [\mathbf{y}_{gps}^i \quad \mathbf{v}_{k+1}^{yz,i}]^T$$

where  $\mathbf{y}_{gps}^i = h(\mathbf{x}_{k+1}^i)$  and  $h(\cdot)$  represents the pseudorange and pseudorange rate measurement models.

## RESULTS AND DISCUSSION

The trajectory data is collected around Seoul National University campus, Seoul, Korea using a low grade IMU and a GPS. The sampling rates are 100 Hz for IMU and 1Hz for GPS respectively. The differential GPS data is used as a reference data in our calculation. The numerical simulations are done in MATLAB using these data for the UKF and the EKF, and the navigation results obtained from UKF is compared with EKF results. Figure 1 shows the difference between position errors in ECEF coordinate for UKF and EKF, and the performance of UKF is slightly better than that of EKF. The small difference

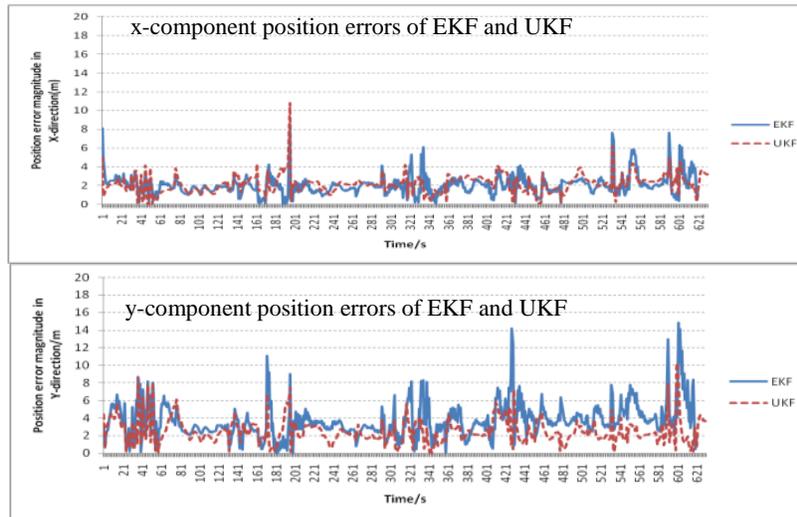
is due to the less non-linearity in land navigations. So, to make the test for robustness, large initial position errors (100m in each X,Y and Z directions) are given to both filters. The responses of the filters are given in Figure 2. In this case UKF has superior performance than EKF.

To test the ability of the UKF algorithm on the GPS outages the satellite's data are rejected in data processing. The results under complete GPS outages are firstly analyzed and then followed by the results under partial GPS outages.

Figure 3 shows the trajectories obtained from different algorithms. The trajectory of the UKF without NHC diverges quickly when the GPS signals are completely blockage while GPS aided trajectory shows good performance. Although the track of UKF with NHC differs from reference trajectory, it keeps its path as possible in clockwise and counter clockwise loops.

Figure 4 illustrates the 3D position errors during the complete GPS outages and each outage is 60 seconds long. The accumulated INS error primarily depends on the grade of IMU used and the time span of the GPS outages. It was found that UKF with NHC has better performance than that without NHC.

The performance of developed UKF with NHC algorithm for partial GPS outage is given in Figure 5. The results clearly show that the NHC significantly improves the navigation performance for periods of signal outage.



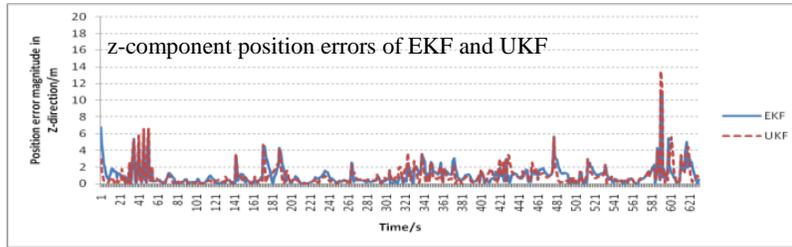


FIGURE 1. Comparison of EKF and UKF in position errors

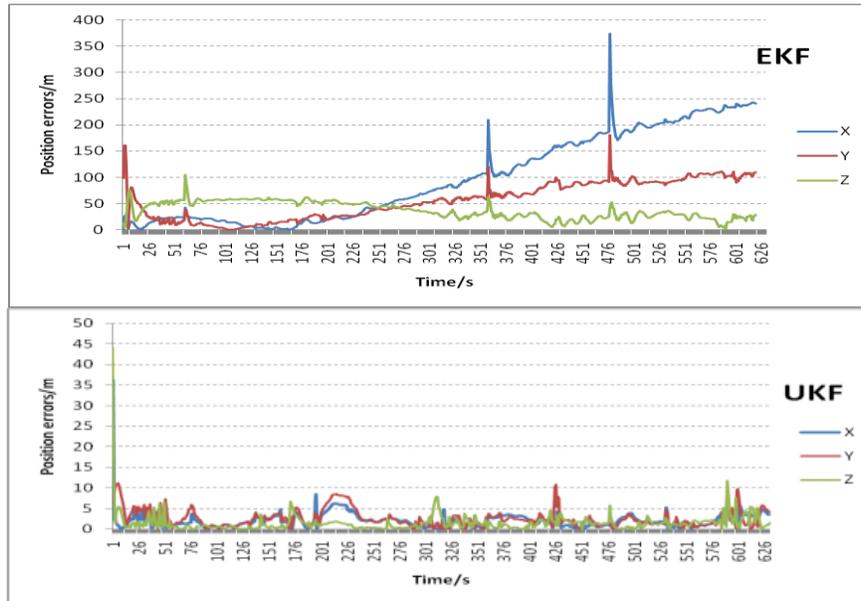


FIGURE 2. Performance of EKF and UKF for large initial position errors

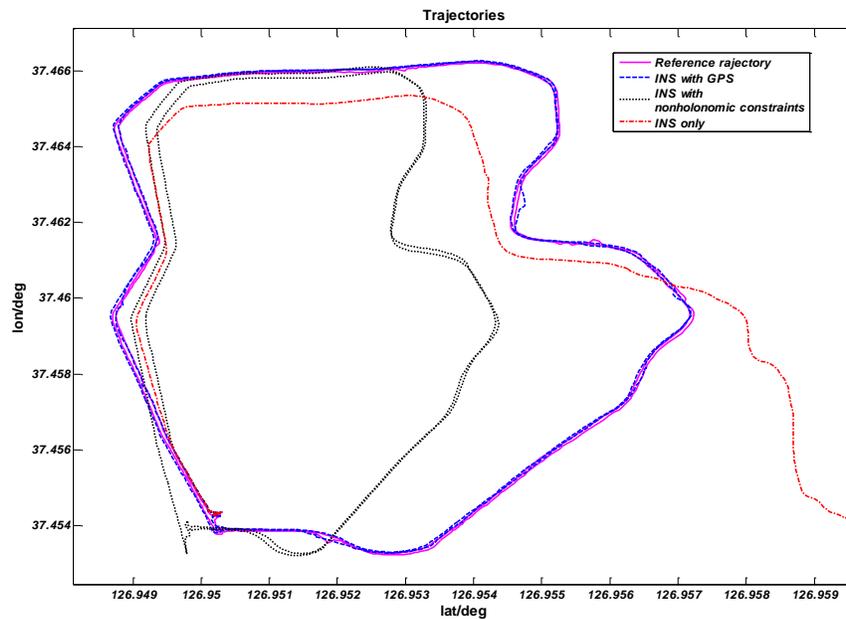


FIGURE 3. Comparison of trajectories

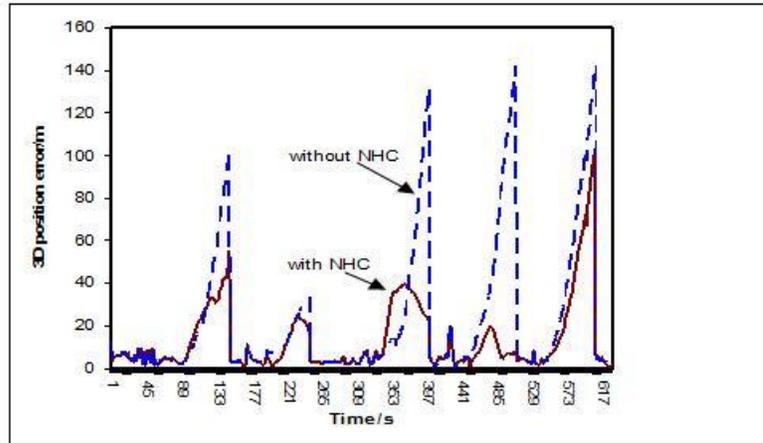
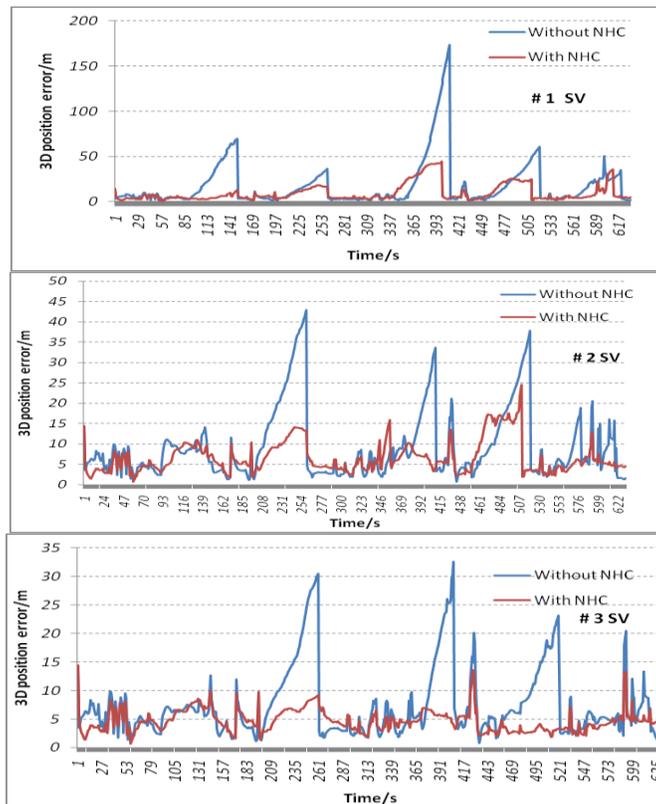


FIGURE 4. 3D position errors under complete GPS outages



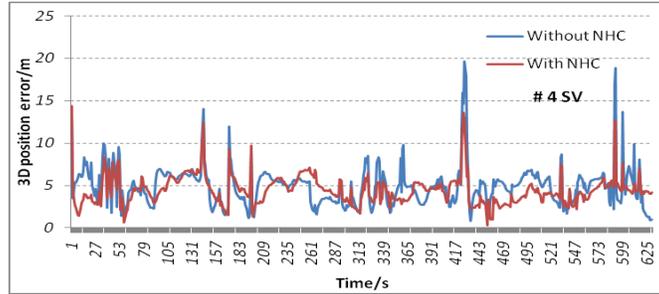


FIGURE 5. 3D position errors under partial GPS outages

## CONCLUSION

In this paper, the UKF is applied to the tightly coupled INS/GPS integration system. To avoid the linearization error, total state model of INS is used instead of error state model. The EKF based INS/GPS integration algorithm is also developed to compare the navigation performance with proposed algorithm. It was found that there is a small difference between EKF and UKF in position solution for land navigation. The significant advantage of our UKF based algorithm was found for extremely large initial position errors. In most of the literatures, the NHCs are used only when the GPS signals are not available. In this work, both NHC and GPS observables are applied simultaneously to the measurement update. The results show that the performance is improved in both completely outage and partial outage conditions of GPS and the average percentage errors are summarized in Table 1. We can conclude that applying UKF on a nonlinear total state model and using NHC with GPS observables do not degrade the performance of the navigation system significantly under the environment with fewer than four satellites and the robustness of the system is improved.

TABLE 1. Average reduction errors

GPS outages	Average reduction error %
No satellite is available	54.4
1 satellite is available	53.2
2 satellites are available	52.6
3 satellites are available	52.4
4 satellites are available	22.6

## REFERENCES

- [1] D. H. Titterton, and J. L. Weston, *Strapdown Inertial Navigation Technology*, 2<sup>nd</sup> Ed., IET and AIAA, 2005.
- [2] S. Kang, *A Design of Deeply-Coupled GPS/INS Integrated Navigation System Using GPS Discriminator Outputs*. Thesis (MS), 2007, Seoul National University, Seoul, Korea.
- [3] S. J. Julier, and J. K. Uhlmann, Unscented Filtering and Nonlinear Estimation, *Proceedings of the IEEE*, 2004, Vol. **92**, pp. 401-422.
- [4] Z. Syed, , P. Aggarwal, Y. Yang, and N. El-Sheimy, Improved Vehicle Navigation Using Aiding with Tightly Coupled Integration. *Proceedings of Vehicular Technology Conference*, Marina Bay, Singapore, 2008, pp. 3077-3081.
- [5] X. Niu, S. Nasser, C. Goodalla1, and N. El-Sheimy, A Universal Approach for Processing any MEMS Inertial Sensor Configuration for Land-Vehicle Navigation. *Journal of Navigation*, 2007, vol. **60**, pp. 233-245.
- [6] K. Kim, and C.G. Park, INS/GPS Tightly Coupled Approach Using an INS Error Predictor. *Proceedings of ION GNSS 18th International Technical Meeting of the Satellite Division*, Long Beach, CA, 2005, pp. 488-493.
- [7] Y. Geng, R. Deurloo, and L. Bastos, Hybrid Dderivative-free Extended Kalman Filter for Unknown Lever Arm Estimation in Tightly Coupled DGPS/INS Integration, *GPS Solutions*, 2010, vol. **57**, pp. 181-191.
- [8] S. Sarkka, On Unscented Kalman Filtering for State Estimation of Continuous-Time Nonlinear Systems, *IEEE Tr. on Automatic Control*, 2007, vol. **52**, pp. 1631-1641.